Ⅳ. Proving

1. Show that the function $f(z)=(z^2-2)e^{-x}e^{-iy}$ is an entire function.
2. Show that if $f^{(m)}(z)\equiv 0(\forall z\in C)$, then $f(z)$ is a polynomial of order $<m$.
3. Show that $\lim\limits_{R\to+\infty}\int_{C_R}\frac{z^2+1}{z^4+5z^2+6}dz=0$, where C_R is the circle centered at 0 with radius R.
4. Suppose that f and $\bar{f}$ are differentiable on a domain D, prove that $f(z)=A$ for some constant A and all $z\in D$.
5. Show that the equation $z^4-4z^2+z-1=0$ has just two roots in the unite disk.
6. Show that $\lim\limits_{R\to+\infty}\int_{C_R}\frac{6z^2+1}{z^4+3z^2+2}dz=0$, where C_R is the circle centered at 0 with radius R.
7. Suppose that f is differentiable and $|f|$ is a constant on a domain D, prove that $f(z)=A$ for some constant A and all $z\in D$.
8. Show that the equation $z^4-7z^3+2z^2+z-1=0$ has just three roots in the unite disk.
9. Show that if $f^{(k)}(z)\equiv 0(\forall z\in C)$, then $f(z)$ is a polynomial of order $<k$.
10. Show that $\lim\limits_{R\to+\infty}\int_{C_R}\frac{7z^2+9}{z^4+7z^2+12}dz=0$, where C_R is the circle centered at 0 with radius R.
11. Show that the equation $z^4-5z^2+2z-1=0$ has just two roots in the unite disk.
12. Show that $\lim\limits_{R\to+\infty}\int_{C_R}\frac{4z^2-1}{z^4+9z^2+20}dz=0$, where C_R is the circle centered at 0 with radius R.
13. Suppose that f is an entire function and there is a constant M and a positive integer m such that $|f(z)|\le M|z|^m$ $(\forall z\in \mathbf{C})$. Prove that

$$f(z)=a_1z+a_2z^2+\cdots+a_mz^m$$

for some constants a_1, $a_2,\cdots,a_m$ and all z in the plane.

14. Show that the equation $z^8-4z^3+z-1=0$ has just three roots in the unite disk.
15. Show that if $f^{(m)}(z)\equiv 0(\forall z\in C)$, then $f(z)$ is a polynomial of order $<m$.

16. Show that $\lim\limits_{R\to+\infty}\int_{C_R}\frac{3z^2+8}{z^4+7z^2+12}dz=0$, where C_R is the circle centered at 0 with radius R.

17. Show that the equation $z^4-5z^2+2z-1=0$ has just two roots in the unite disk.

18. Show that $\lim\limits_{R\to+\infty}\int_{C_R}\frac{5z^2-7}{z^4+9z^2+20}dz=0$, where C_R is the circle centered at 0 with radius R.

19. Suppose that f is an entire function and there is a constant M and a positive integer m such that $|f(z)|\le M|z|^m$ $(\forall z\in \mathbf{C})$. Prove that

$$f(z)=a_1z+a_2z^2+\cdots+a_mz^m$$

for some constants a_1, $a_2,\ldots,a_m$ and all z in the plane.

20. Show that the equation $z^8=4z^3-z+1$ has just three roots in the unite disk.

21. Show that $\lim\limits_{R\to+\infty}\int_{C_R}\frac{6z^2+1}{z^4+3z^2+2}dz=0$, where C_R is the circle centered at 0 with radius R.

22. Suppose that f is differentiable and $|f|$ is a constant on a domain D, prove that $f(z)=A$ for some constant A and all $z\in D$.

23. Show that the equation $z^4-7z^3+2z^2+z-1=0$ has just three roots in the unite disk.

24. Show that $\lim\limits_{R\to+\infty}\int_{C_R}\frac{z^2+3}{z^4+3z^2+2}dz=0$, where C_R is the circle centered at 0 with radius R.

25. Show that if $f^{(m)}(z)\equiv 0(\forall z\in C)$, then $f(z)$ is a polynomial of order $<m$.

26. Show that $\lim\limits_{R\to+\infty}\int_{C_R}\frac{3z^2+8}{z^4+7z^2+12}dz=0$, where C_R is the circle centered at 0 with radius R.

27. Show that the equation $z^4-5z^2+2z-1=0$ has just two roots in the unite disk.