Ⅲ. Computations

1. Find $\int_{|z|=1} \frac{zdz}{(2z+1)(z-2)}$.

2. Find the value of $\int_{|z|=1} \frac{e^z \sin^2 z}{z^2} dz + \int_{|z|=2} \frac{z^3 dz}{(1-z)^2}$.

3. Let $f(z) = \frac{z}{(z-1)(z-2)}$, find the Laurent expansion of f on the annulus $D = \{z : 0 < |z| < 1\}$.

4. Given $f(z) = \int_C \frac{2\lambda^2 + 4\lambda + 1}{\lambda - z} d\lambda$, where $C = \{z : |z| = 3\}$, find $f'(-1+i)$.

5. Given $f(z) = \frac{\sin^2 z}{(z-1)(z+1)}$, find $\mathrm{Res}(f(z),1) + \mathrm{Res}(f(z),-1)$

6. Find $\int_{|z|=1} \frac{zdz}{(3z+1)(3z+2)}$.

7. Find the value of $\int_{|z|=1} \frac{z - \sin z}{z^6} dz + \int_{|z|=2} \frac{z^2 dz}{(z^2-1)}$.

8. Let $f(z) = \frac{z^2}{(z+1)(z+2)}$, find the Laurent expansion of f on the annulus $D = \{z : 0 < |z| < 1\}$.

9. Given $f(z) = \int_C \frac{3\xi^2 + 4\xi + 1}{\xi - z} d\xi$, where $C = \{z : |z| = 4\}$, find $f'(2+i)$.

10. Evaluate $\mathrm{Res}(\frac{z^2}{(z^2+1)^2}, i)$.

11. Find $\int_{|z|=1} \frac{5zdz}{(2z+1)(z-2)}$.

12. Find the value of $\int_{|z|=1} \frac{e^z \sin^2 z}{z^2} dz + \int_{|z|=2} \frac{z^8 dz}{(1-z)^2}$.

13. Let $f(z) = \frac{z}{(z-1)(z-2)}$, find the Laurent expansion of f on the annulus $D = \{z : 0 < |z| < 1\}$.

14. Given $f(z) = \int_C \frac{5\lambda^2 + 4\lambda + 3}{\lambda - z} d\lambda$, where $C = \{z : |z| = 3\}$, find $f'(-1+i)$.

15. Given $f(z)=\dfrac{1+\sin^2 z}{(z-1)(z+1)}$, find $\mathrm{Res}(f(z),1)+\mathrm{Res}(f(z),-1)$.

16. Find the integral $\int_C \dfrac{e^z}{z^2+1}dz$, where C is the circle $|z|=7$.

17. Find the value of $\int_{|z|=1} \dfrac{e^{z+1}\cos z}{z^2}dz+\int_{|z|=2}\dfrac{z^5dz}{(1-z)^3}$.

18. Let $f(z)=\dfrac{1}{(z-1)(z-2)}$, find the Laurent expansion of f on the annulus $D=\{z:0<|z|<1\}$.

19. Given $f(z)=\int_C \dfrac{5\lambda^2+6\lambda+7}{\lambda-z}d\lambda$, where $C=\{z:|z|=4\}$, find $f'(1+i)$.

20. Given $f(z)=\dfrac{z+2}{z}, C: z=2e^{i\theta}(0\le\theta\le\pi)$, find $\int_C f(z)dz$.

21. If a function f is differentiable at a point z_0, then it is analytic at z_0.（ ）

22. If a point z_0 is a pole of order k of f, then z_0 is a zero of order k of $1/f$.（ ）

23. A bounded entire function must be a constant.（ ）

24. A function f is analytic a point $z_0=x_0+iy_0$ if and only if whose real and imaginary parts are differentiable and the Cauchy Riemann conditions hold in a neighborhood of (x_0,y_0).（ ）

25. If a function f is continuous on the plane and $\int_C f(z)dz=0$ for every simple closed contour C, then $f(z)+e^z\sin z$ is an entire function.（ ）

26. Find $\int_{|z|=1}\dfrac{zdz}{(2z+1)(z-2)}$.

27. Find the value of $\int_{|z|=1} \dfrac{e^z\sin^2 z}{z^2}dz+\int_{|z|=2}\dfrac{z^3dz}{(1-z)^2}$.

28. Let $f(z)=\dfrac{z}{(z-1)(z-2)}$, find the Laurent expansion of f on the annulus $D=\{z:0<|z|<1\}$.

29. Given $f(z)=\int_C \frac{2\lambda^2+4\lambda+1}{\lambda-z}d\lambda$, where $C=\{z:|z|=3\}$, find $f'(-1+i)$.

30. Given $f(z)=\frac{\sin^2 z}{(z-1)(z+1)}$, find $\text{Res}(f(z),1)+\text{Res}(f(z),-1)$.

31. Find the integral $\int_C \frac{e^z+z^2}{z^2+1}dz$, where C is the circle $|z|=7$.

32. Find the value of $\int_{|z|=1}\frac{e^{z+1}\cos z}{z^2}dz+\int_{|z|=2}\frac{z^5dz}{(1-z)^3}+\int_{|z|=3}\cos^{2007} z dz$.

33. Let $f(z)=\frac{1}{(z-1)(z-2)}$, find the Laurent expansion of f on the annulus $D=\{z:0<|z|<1\}$.

34. Given $f(z)=\int_C \frac{9\lambda^2+6\lambda+7}{\lambda-z}d\lambda$, where $C=\{z:|z|=5\}$, find $f'(1+i)$.

35. Given $f(z)=\frac{3z+2}{z}, C: z=2e^{i\theta}(0\le\theta\le\pi)$, find $\int_C f(z)dz$.

36. Find $\int_{|z|=1}\frac{zdz}{(3z+1)(3z+2)}$.

37. Find the value of $\int_{|z|=1}\frac{z-\sin z}{z^6}dz+\int_{|z|=2}\frac{z^2dz}{(z^2-1)}$.

38. Let $f(z)=\frac{z^2}{(z+1)(z+2)}$, find the Laurent expansion of f on the annulus $D=\{z:0<|z|<1\}$.

39. Given $f(z)=\int_C \frac{3\xi^2+4\xi+1}{\xi-z}d\xi$, where $C=\{z:|z|=4\}$, find $f'(2+i)$.

40. Evaluate $\text{Res}(\frac{z^2}{(z^2+1)^2},i)$.

41. Find $\int_{|z|=1}\frac{9zdz}{(3z+1)(3z+2)}$.

42. Find the value of $\int_{|z|=1}\frac{z^2-\sin z}{z^4}dz+\int_{|z|=2}\frac{z^2dz}{(z^2-1)}$.

43. Let $f(z)=\frac{3z^2}{(z+1)(z+2)}$, find the Laurent expansion of f on the annulus

$D = \{z : 0 < |\, z \,| < 1\}$.

44. Given $f(z) = \int_C \frac{3\xi^2 + 4\xi + 5}{\xi - z} d\xi$, where $C = \{z : |\, z \,| = 4\}$, find $f'(2+i)$.

45. Find $\operatorname{Res}\left(\frac{4z^2}{(z^2+1)^2}, i \right)$.

46. Find $\int_{|z|=1} \frac{z dz}{(2z+1)(z-2)}$.

47. Find the value of $\int_{|z|=1} \frac{e^z \sin^2 z}{z^2} dz + \int_{|z|=2} \frac{z^3 dz}{(1-z)^2}$.

48. Let $f(z) = \frac{z}{(z-1)(z-2)}$, find the Laurent expansion of f on the annulus $D = \{z : 0 < |\, z \,| < 1\}$.

49. Given $f(z) = \int_C \frac{2\lambda^2 + 4\lambda + 1}{\lambda - z} d\lambda$, where $C = \{z : |\, z \,| = 3\}$, find $f'(-1+i)$.

50. Given $f(z) = \frac{\sin^2 z}{(z-1)(z+1)}$, find $\operatorname{Res}(f(z), 1) + \operatorname{Res}(f(z), -1)$.