Exercises 5.1

1. Write $z = re^{i\theta}$, where $0 < r < 1$ and θ is real, (5.1.9) Then, with the aid of Theorem 5.1.1 show that

$$\sum_{n=1}^{\infty} r^n \cos n\theta = \frac{r\cos\theta - r^2}{1 - 2r\cos\theta + r^2} \quad \text{and} \quad \sum_{n=1}^{\infty} r^n \sin n\theta = \frac{r\sin\theta}{1 - 2r\cos\theta + r^2}$$

when $0 < r < 1$. (Note that these formulas are also valid when $r = 0$.)

2. Show that if $\sum_{n=1}^{\infty} z_n = S$, then $\sum_{n=1}^{\infty} \overline{z}_n = \overline{S}$.

3. Show that if $\sum_{n=1}^{\infty} z_n = S$, then $\sum_{n=1}^{\infty} cz_n = cS, \forall c \in \mathbf{C}$.