§2.13. Analytic Functions

We are now ready to introduce the concept of an analytic function.

1.Definition of an analylic function

A function f of the complex variable z is *analytic* in an open set D if it has a derivative at each point in D. A function f is *analytic in a set* S if it is analytic in an open set containing S. In particular, f is *analytic at a point* z_0 if it is analytic in some neighborhood of z_0.

2. Definition of an entire function

An *entire function* is a function that is analytic at each point in the entire finite plane.

3.definition of singular point

If a function f fails to be analytic at a point z_0 but is analytic at some point in every neighborhood of z_0, then z_0 is called a *singular point*, or *singularity*, of f.

Proposition 2.12.1. *If two functions* f, g *are analytic in a domain* D*, then* $f \pm g, fg$ *are analytic in* D*. And,* f / g *is analytic in* D *provided that* $g(z) \neq 0$ *at any point in* D.

Proposition 2.12.2. *S*uppose that f and g are analytic in a domain D and G, respectively, with $f(D) \subset G$, then the composition $g \circ f$ is analytic in D with derivative

$$\frac{d}{dz} g[f(z)] = g'[f(z)] f'(z), \forall z \in D.$$

Theorem 2.13.1. *If* $f'(z) = 0$ *everywhere in a domain* D*, then* $f(z)$ *must be constant throughout* D.