## §2.9. Differentiation Formulas

The definition of derivative in Sec. 2.8 is identical in form to that of the derivative of a real-valued function of a real variable. In fact, the basic differentiation formulas given below can be derived from that definition by essentially the same steps as the ones used in calculus. In these formulas, the derivative of a function $f$ at a point $z$ is denoted by either $\frac{d}{dz}f(z)$ or $f'(z)$, depending on which notation is more convenient.

**1. Basic equations**

Let $c$ be a complex constant, and let $f$ be a function whose derivative exists at a point $z$. It is easy to show that

$$\frac{d}{dz}c = 0, \quad \frac{d}{dz}z = 1, \quad \frac{d}{dz}[cf(z)] = cf'(z). \tag{2.9.1}$$

Also, if $n$ is a positive integer, then

$$\frac{d}{dz}z^n = nz^{n-1}. \tag{2.9.2}$$

This formula remains valid when $n$ is a negative integer, provided that $z \neq 0$.

**2. Theorems**

**Theorem 2.9.1.** *If two function $f$ and $g$ are differentiable at a point $z$, then*

$$\frac{d}{dz}[f(z)+g(z)] = f'(z)+g'(z), \tag{2.9.3}$$

$$\frac{d}{dz}[f(z)g(z)] = f(z)g'(z)+f'(z)g(z); \tag{2.9.4}$$

*and, when $g(z) \neq 0$,*

$$\frac{d}{dz}\left[\frac{f(z)}{g(z)}\right] = \frac{g(z)f'(z)-f(z)g'(z)}{[g(z)]^2}. \tag{2.9.5}$$

**Theorem 2.9.2.** *Suppose that $f$ has a derivative at $z_0$ and that $g$ has a derivative at the point $f(z_0)$. Then the function $F(z) = g(f(z))$ has a derivative at $z_0$, and*

$$F'(z_0) = g'(f(z_0))f'(z_0). \tag{2.9.6}$$

**Example**. To find the derivative of $(2z^2+i)^5$, write $w = 2z^2+i$ and $W = w^5$. Then

$$\frac{d}{dz}(2z^2+i)^5 = 5w^4 4z = 20z(2z^2+i)^4.$$