§2.7. Continuity

1. Definition of a continuous function

A function $f : D \to \mathbf{C}$ is called *continuous* at z_0 if

$$\lim_{z \to z_0} f(z) = f(z_0). \tag{2.7.1}$$

A function $f : D \to \mathbf{C}$ is said to be *continuous on* D if it is continuous at each point in D.

2. Theorems

Theorem 2.7.1. *Let* $w = f(z)$ *be a function defined on some neighborhood* $N(z_0, r)$ *of a point* z_0*, and* $W = g(w)$ *be a function defined on* G *such that* $f(N(z_0, r)) \subset G$*. If* f *is continuous at* z_0 *and* g *is continuous at the point* $f(z_0)$*, then the composition* $W = g[f(z)]$ *is continuous at* z_0*.*

Theorem 2.7.2. *If a function* f *is continuous and nonzero at an interior point* z_0 *of the domain of definition, then* $f(z) \neq 0$ *throughout some neighborhood of* z_0*.*

Theorem 2.7.3. *Suppose that the function* (2.7.3) *is continuous on a closed and bounded region* R*, then* f *is bounded on* R *and* $|f(z)|$ *reaches a maximum value somewhere in* R*.*